On the decay of soliton excitations

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Abstract
In field theory the scattering about spatially extended objects, such as solitons, is commonly described by small amplitude fluctuations. Since soliton configurations often break internal symmetries, excitations exist that arise from quantizing the modes that are introduced to restore these symmetries. These modes represent collective distortions and cannot be treated as small amplitude fluctuations. Here, we present a method to embrace their contribution to the scattering matrix. In essence, this allows us to compute the decay widths of such collective excitations. As an example, we consider the Skyrme model for baryons and explain that the method helps to solve the long-standing Yukawa problem in chiral soliton models.

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1. Statement of the problem

Phase shifts, δ, are essential tools to probe external forces in field theory because—among other applications—they describe the response to background potentials. In turn they play an important role for determining Casimir (or vacuum polarization) energies,

$$E_{\text{vac}} \sim \int \frac{\mathrm{d}k}{2\pi} \omega_k \frac{\mathrm{d}}{\mathrm{d}k} \delta(k) + E_{\text{c.t.}}, \tag{1}$$

of extended objects, such as solitons Φ_{cl}. Usually, these phase shifts are computed from small amplitude fluctuations about Φ_{cl}. Solitons often break symmetries of the fundamental theory. For example, the Skyrme hedgehog soliton is not rotationally invariant. This applies to both coordinate and flavor rotations. In turn this gives rise to large amplitude fluctuations in these directions. The symmetries are restored by canonically quantizing collective coordinates that parameterize the orientation of the soliton in the corresponding spaces. The so-constructed states correspond to resonances of the soliton and their Yukawa exchanges significantly contribute to the scattering data. Yet, this important contribution is not necessarily captured by the small amplitude fluctuations. Here, we will exemplify their incorporation within the Skyrme model.

There is a serious problem for computing the properties of resonances soliton models. Commonly, the coupling of resonances to mesons is described by a Yukawa interaction of the generic structure

$$\Gamma_{\text{int}}[\psi_{B'}, \psi_B, \phi] \sim g \int \mathrm{d}^4 x \, \bar{\psi}_{B'} \phi \psi_B, \tag{2}$$

where B' is the resonance that might decay into the (ground) state B and meson ϕ and g is a coupling constant. It is crucial that this interaction is *linear* in ϕ! If ϕ is pseudoscalar, this interaction yields the decay width (residuum of the scattering amplitude) $\Gamma(B' \to B\phi) \propto g^2 |\vec{p}_\phi|^3$, with $\vec{p}_\phi$ being the momentum of the outgoing meson. Soliton models, however, are based on action functionals of *only* meson degrees of freedom, $\Gamma = \Gamma[\Phi]$. They contain classical (static) soliton solutions, Φ_{sol}, that are identified as baryons whose interaction with the mesons is described by the (small) meson fluctuations about the soliton: $\Phi = \Phi_{\text{sol}} + \phi$. By pure definition of the stationary condition, the expansion of $\Gamma[\Phi]$ about Φ_{sol} does not have a term that is linear in ϕ to be interpreted as Yukawa interaction, equation (2). This puzzle has become famous as the Yukawa problem in soliton models. Hence, the resonance properties must be extracted from meson baryon scattering amplitudes. In soliton models two-meson processes acquire contributions from the second-order term

$$\Gamma^{(2)} = \frac{1}{2} \phi \cdot \left. \frac{\delta^2 \Gamma[\Phi]}{\delta^2 \Phi} \right|_{\Phi = \Phi_{\text{sol}}} \cdot \phi. \tag{3}$$

This also represents an expansion in N, the number of internal degrees of freedom (color in strong interactions): $\Gamma = \mathcal{O}(N)$ and $\Gamma^{(2)} = \mathcal{O}(N^0)$. Terms $\mathcal{O}(\phi^3)$ vanish when $N \to \infty$. Thus, $\Gamma^{(2)}$ contains all large-N information about the contribution of resonances to scattering data.

The large-N expansion is systematic but a low order truncation is not necessarily reliable at the physical point and it is very challenging to reliably compute subleading contributions. Presumably resonance exchanges contribute significantly in that regime. To probe the reliability of the computed resonance contributions we transform the above statement into a consistency condition: for $N \to \infty$ any valid computation of hadronic decay widths in soliton models *must* identically match the result obtained from $\Gamma^{(2)}$. Unfortunately, the most prominent baryon resonance, the Δ isobar, becomes degenerate with the nucleon as $N \to \infty$. It is stable in that limit and its decay is not subject to the just described litmus test. The situation is more interesting in soliton models for flavor $SU(3)$. In the so-called rigid rotator approach (RRA), that generates baryon states as (flavor) rotational excitations of the soliton, exotic resonances emerge that dwell in the anti-decuplet representation of flavor $SU(3)$ [1]. The most discussed (and disputed) such state is the Θ^+ pentaquark with zero isospin and strangeness $S = +1$. When $N \to \infty$ the mass difference between anti-decuplet states and the nucleon does not vanish. So the properties of Θ^+ predicted from any model treatment must also be seen in the quantizing of the strangeness degrees of freedom based on the harmonic approximation, $\Gamma^{(2)}$. This (seemingly alternative) quantization is called the bound state approach (BSA) for reasons that will become obvious later. The above-discussed litmus test requires that the BSA and RRA give identical results for Θ^+ as $N \to \infty$. This did not seem to be true and it was argued that the prediction of pentaquarks would be a mere artifact of the RRA [2]. We will show that this conclusion is premature. Furthermore, the comparison between the BSA and RRA provides an unambiguous computation of pentaquark widths: it differs substantially from approaches [3] that identified the transition operators for $\Theta^+ \to KN$ from the axial current.

This presentation is based on [4] which should be consulted for further details.

2. The model

For simplicity, we consider the Skyrme model [5] as a particular example for chiral soliton models. However, we stress that our qualitative results generalize to *all* chiral soliton models because these results solely reflect the treatment of the model degrees of freedom.

Chiral soliton models are functionals of the chiral field, U, the nonlinear realization of the pseudoscalar mesons[1], ϕ_a,

$$U(\vec{x}, t) = \exp\left[\frac{\mathrm{i}}{f_\pi}\phi_a(\vec{x}, t)\lambda_a\right], \tag{4}$$

with λ_a being the Gell–Mann matrices of $SU(3)$. We split the action into three pieces $\Gamma = \Gamma_{\mathrm{SK}} + \Gamma_{\mathrm{WZ}} + \Gamma_{\mathrm{SB}}$. The first term represents the Skyrme model action,

$$\Gamma_{\mathrm{SK}} = \int \mathrm{d}^4x \, \mathrm{tr}\left\{\frac{f_\pi^2}{4}[\partial_\mu U \partial^\mu U^\dagger] + \frac{1}{32\epsilon^2}[[U^\dagger\partial_\mu U, U^\dagger\partial_\nu U]^2]\right\}. \tag{5}$$

Here $f_\pi = 93\,\mathrm{MeV}$ is the pion decay constant and ϵ is the Skyrme parameter. The two-flavor version of the Skyrme model suggests $\epsilon = 4.25$ to reproduce the Δ-nucleon mass difference[2]. The QCD anomaly is incorporated via the Wess–Zumino action [7],

$$\Gamma_{\mathrm{WZ}} = -\frac{\mathrm{i}N}{240\pi^2} \int \mathrm{d}^5x \epsilon^{\mu\nu\rho\sigma\tau} \, \mathrm{tr}[\alpha_\mu\alpha_\nu\alpha_\rho\alpha_\sigma\alpha_\tau], \tag{6}$$

with $\alpha_\mu = U^\dagger\partial_\mu U$. The flavor symmetry breaking terms are contained in Γ_{SB}:

$$\Gamma_{\mathrm{SB}} = \frac{f_\pi^2}{4} \int \mathrm{d}^4x \, \mathrm{tr}[\mathcal{M}(U + U^\dagger - 2)], \qquad \mathcal{M} = \mathrm{diag}\left(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2\right). \tag{7}$$

We do not include terms that distinguish between pion and kaon decay constants even though they differ by about 20% empirically. This omission is a matter of convenience and leads to an underestimation of symmetry breaking effects [8] which approximately can be accounted for by rescaling the kaon mass $m_K \to m_K f_K/f_\pi$. The action has a topologically non-trivial classical solution, the famous hedgehog soliton

$$\Phi_{\mathrm{sol}} \sim U_0(\vec{x}) = \exp[\mathrm{i}\vec{\lambda} \cdot \hat{x} F(r)], \qquad r = |\vec{x}| \tag{8}$$

embedded in the isospin subspace of flavor $SU(3)$. The chiral angle, $F(r)$, solves the classical equation of motion subject to the boundary condition $F(0) - F(\infty) = \pi$ ensuring unit winding (baryon) number. In the RRA baryon states are generated by canonically quantizing collective coordinates $A \in SU(3)$ that describe the (spin) flavor orientation of the soliton, $A(t)U_0(\vec{x})A^\dagger(t)$. The resultant eigenstates may be classified according to $SU(3)$ multiplets, see [9] for a review.

3. Small amplitude fluctuations in the P-wave channel with strangeness

As motivated in section 1, we introduce fluctuations $\phi \sim \eta_\alpha(\vec{x}, t)$,

$$U(\vec{x}, t) = \sqrt{U_0(\vec{x})} \exp\left[\frac{\mathrm{i}}{f_\pi}\lambda_\alpha\eta_\alpha(\vec{x}, t)\right]\sqrt{U_0(\vec{x})}, \tag{9}$$

for the kaon fields [10]. Expanding the action in powers of these fluctuations yields $\Gamma^{(2)}$ at first non-zero order. The P-wave mode is characterized by a single radial function

$$\begin{pmatrix} \eta_4 + \mathrm{i}\eta_5 \\ \eta_6 + \mathrm{i}\eta_7 \end{pmatrix}_P (\vec{x}, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}\omega t}\eta(r, \omega)\hat{x} \cdot \vec{\tau}\chi(\omega). \tag{10}$$

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[1] Repeated indices are summed as $a, b, c, \ldots = 1, \ldots, 8$, $\alpha, \beta, \gamma, \ldots = 4, \ldots, 7$ and $i, j, k, \ldots = 1, 2, 3$.
[2] To ensure that the (perturbative) n-point functions scale as $N^{1-n/2}$ [6] we substitute $f_\pi = 93\,\mathrm{MeV}\sqrt{N/3}$ and $\epsilon = 4.25\sqrt{3/N}$ in the study of the N dependence.
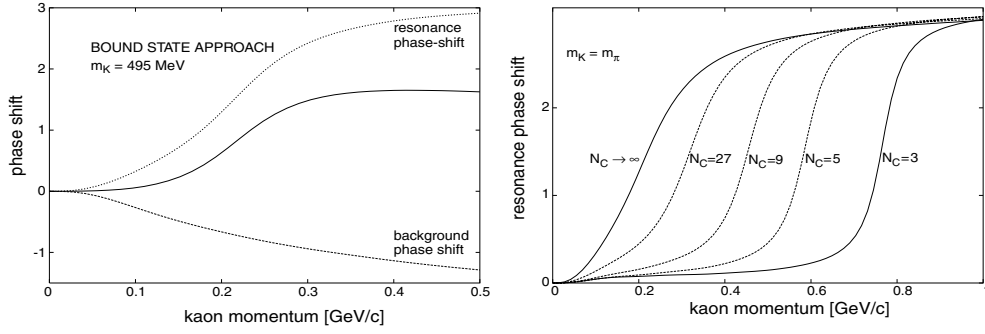
Figure 1. Left panel: large-*N* P-wave phase shifts with strangeness $S = +1$. The full line shows the phase extracted from equation (11). Background and resonance phase shifts are defined in the text. Right panel: the resonance phase shift for various N and $m_K = m_\pi$.

In future we will omit the argument for the Fourier frequency. Upon quantization the components of the two-component iso-spinor χ are elevated to creation and annihilation operators. It is straightforward to deduce the Schrödinger-type equation

$$h^2\eta(r) + \omega\left[2\lambda(r) - \omega M_K(r)\right]\eta(r) = 0 \qquad \text{with} \qquad h^2 = -\frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} + V_{\text{eff}}(r). \quad (11)$$

The radial functions arise from the chiral angle $F(r)$ and may be taken from the literature [10]. The equation of motion (11) is not invariant under particle conjugation $\omega \leftrightarrow -\omega$, and thus different for kaons ($\omega > 0$) and anti-kaons ($\omega < 0$). This difference is caused by $\lambda(r) \neq 0$ which originates from Γ_{WZ}. Equation (11) has a bound state solution (hence the notion *bound state approach*) at $\omega = -\omega_\Lambda$ which equals the mass difference between the Λ-hyperon and the nucleon in the large-*N* limit. As this energy eigenvalue is negative it corresponds to a kaon, i.e. it carries strangeness $S = -1$. In the symmetric case ($m_K = m_\pi$) the bound state is the zero mode of $SU(3)$ flavor symmetry. Since Γ_{WZ} moves the potential bound state with $S = +1$ to $\omega_\Theta > m_K$ we expect a resonance structure in that channel. The corresponding phase shift is shown in the left panel of figure 1. No clear resonance structure is visible; the phase shifts hardly reach $\pi/2$. The absence of such a resonance has previously led to the premature criticism that there would not exist a bound pentaquark in the large-*N* limit [2].

4. Constraint fluctuations in the flavor symmetric case

We couple the fluctuations to the collective excitations by generalizing equation (9) to

$$U(\vec{x}, t) = A(t)\sqrt{U_0(\vec{x})}\exp\left[\frac{\mathrm{i}}{f_\pi}\lambda_\alpha\widetilde{\eta}_\alpha(\vec{x}, t)\right]\sqrt{U_0(\vec{x})}A^\dagger(t). \qquad (12)$$

These fluctuations dwell in the intrinsic system as they rotate along with the soliton. The kaon P-wave is subject to the modified integro-differential equation

$$h^2\widetilde{\eta}(r) + \omega[2\lambda(r) - \omega M_K(r)]\widetilde{\eta}(r) = -z(r)\left[\int_0^\infty r'^2\,\mathrm{d}r'z(r')2\lambda(r')\widetilde{\eta}(r')\right]$$

$$\times\left[2\lambda(r) - (\omega + \omega_0)M_K(r) - \omega_0\left(\frac{X_\Theta^2}{\omega_\Theta - \omega} + \frac{X_\Lambda^2}{\omega}\right)(2\lambda(r) - \omega_0 M_K(r))\right], \qquad (13)$$

for the flavor symmetric case[3]. The radial function $\widetilde{\eta}(r)$ is defined according to equation (10) and $z(r) = \sqrt{4\pi}\,\frac{f_\pi}{\sqrt{\Theta_K}}\sin\frac{F(r)}{2}$ is the collective mode wavefunction normalized with respect to the moment of inertia for flavor rotations into the strangeness direction, $\Theta_K = f_\pi^2 \int \mathrm{d}^3 r\, M_K(r)\sin^2\frac{F(r)}{2} = \mathcal{O}(N)$. The non-local terms reflect the constraint $\int \mathrm{d}r\, r^2 z(r) M_K(r)\widetilde{\eta}(r) = 0$ which avoids double counting of rotational modes in the strangeness direction. The interesting coupling is

$$H_{\text{int}} = \frac{2}{\sqrt{4\pi\,\Theta_K}} d_{i\alpha\beta} D_{\gamma\alpha} R_\beta \int \mathrm{d}^3 r\, z(r)\left[2\lambda(r) - \omega_0 M_K(r)\right]\hat{x}_i \widetilde{\xi}_\gamma(\vec{x}, t), \qquad (14)$$

where $\widetilde{\xi}_a = D_{ab}\widetilde{\eta}_b$ are the fluctuations in the laboratory frame that we actually detect in KN scattering. The collective coordinates are parameterized via the adjoint representation $D_{ab}(A) = \frac{1}{2}\,\mathrm{tr}[\lambda_a A \lambda_b A^\dagger]$ and the $SU(3)$ generators R_a. Integrating out the collective degrees of freedom induces the separable potential

$$\frac{|\langle\Theta|H_{\text{int}}|(KN)_{I=0}\rangle|^2}{\omega_\Theta - \omega} + \frac{|\langle\Lambda|H_{\text{int}}|(KN)_{I=0}\rangle|^2}{\omega_\Lambda + \omega}. \qquad (15)$$

These matrix elements concern the T-matrix elements in the laboratory frame. For the Θ^+ channel it is identical to the one in the intrinsic system [11]. Thus we may directly add the exchange potential, equation (15), to the Hamiltonian for the intrinsic fluctuations. We define matrix elements of collective coordinate operators,

$$\langle\Theta^+|d_{3\alpha\beta} D_{+\alpha} R_\beta|n\rangle =: X_\Theta\sqrt{\frac{N}{32}} \qquad \text{and} \qquad \langle\Lambda|d_{3\alpha\beta} D_{-\alpha} R_\beta|p\rangle =: X_\Lambda\sqrt{\frac{N}{32}}, \qquad (16)$$

to end up with equation (13). The first factor in the coefficient $\omega_0 = 2\left(\frac{2}{\sqrt{\Theta_K}}\sqrt{\frac{N}{32}}\right)^2 = \frac{N}{4\Theta_K}$ arises in the equation of motion because the potential, equation (15), is quadratic in the fluctuations. The remaining (squared) factors stem from the definitions of $X_{\Theta,\Lambda}$ and the constant of proportionality in H_{int}. $X_{\Theta,\Lambda}$ must be computed with the methods provided in [12] but generalized to arbitrary (odd) N [4]. For $N \to \infty$ we have $X_\Theta \to 1$ and $X_\Lambda \to 0$. From the orthogonality conditions of the equation of motion (11) we straightforwardly verify that in this limit

$$\widetilde{\eta}(r) = \eta(r) - a z(r) \qquad \text{with} \qquad a = \int_0^\infty \mathrm{d}r\, r^2 z(r) M_K(r)\eta(r) \qquad (17)$$

solves equation (13). This is essential because, as $z(r)$ is localized in space, η and $\widetilde{\eta}$ have identical phase shifts! Hence, the large-N consistency condition discussed in the introduction is indeed satisfied. The physics becomes more transparent when considering the background wavefunction $\overline{\eta}(r)$ that solves equation (13) for $X_\Theta \equiv X_\Lambda \equiv 0$, i.e. the collective excitations are decoupled. We stress that $\overline{\eta}(r)$ is a purely large-N quantity that may be obtained by demanding the BSA wavefunction to be orthogonal to the collective mode. In doing so, the asymptotic behavior of $\overline{\eta}(r)$ gives the background phase shift shown in figure 1. Subsequently, we may again switch on the exchange contributions, equation (15). The additional separable potential augments the phase shift by

$$\tan\left(\delta_R(\omega)\right) = \frac{\Gamma(\omega_k)/2}{\omega_\Theta - \omega + \Delta(\omega)}. \qquad (18)$$

Here, $\omega_\Theta = \frac{N+3}{4\Theta_K}$ is the RRA result for the excitation energy of Θ. This phase shift exhibits the canonical resonance structure with the width and the pole shift

$$\Gamma(\omega) = 2k\omega_0 X_\Theta^2 \left|\int_0^\infty r^2\,\mathrm{d}r\, z(r) 2\lambda(r)\overline{\eta}(r, \omega)\right|^2, \qquad (19)$$

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[3] The more complicated case $m_K \neq m_\pi$ is at length discussed in [4].

$$\Delta(\omega) = \frac{1}{2\pi\omega}\mathcal{P}\int_0^\infty q\, dq\left[\frac{\Gamma(\omega_q)}{\omega - \omega_q} + \frac{\Gamma(-\omega_q)}{\omega + \omega_q}\right] \qquad \left(\omega_q = \sqrt{q^2 + m_K^2}\right). \tag{20}$$

We have numerically verified that in the large-N limit with $X_\Theta^2 = 1$, the phase shift from equation (18) is identical to what is labeled resonance phase shift in figure 1 that we calculated as the difference between the total (η) and background ($\overline{\eta}$) phase shifts. It stems from a completely different computation: while all phase shifts shown in figure 1 result solely from large-N computations, equation (18) yields an N dependent phase shift that for $N \to \infty$ reproduces the resonance phase shift. This clearly shows that contrary to earlier criticisms [2] the large-N pentaquark channel indeed resonates! It furthermore suggests that the small amplitude approach (3) might give insufficient results for scattering data.

5. Results

In figure 1, we show the resonance phase shift computed from equation (18) for various values of N. First we observe that the resonance position quickly moves toward larger energies as N decreases. This is mainly due to the strong N dependence of ω_Θ: for $N = 3$ it is twice as large as in the limit $N \to \infty$. The pole shift Δ is quite small (some 10 MeV) so ω_Θ is indeed a reliable estimate of the resonance energy. Second, the resonance becomes shaper as N decreases. To discuss the quantitative results we now include flavor symmetry breaking effects. Then, the resonance position changes to

$$\omega_\Theta = \frac{1}{2}\left[\sqrt{\omega_0^2 + \frac{3\Gamma}{2\Theta_K}} + \omega_0\right] + \mathcal{O}\left(\frac{1}{N}\right), \tag{21}$$

where $\Gamma = \mathcal{O}(N)$ is a functional of the soliton that is proportional to the meson mass difference, $m_K^2 - m_\pi^2$. The $\mathcal{O}(1/N)$ piece is sizable for $N = 3$ and we compute it in the scenario of [12]. We then find $\omega_\Theta \approx 700$ MeV; taking model dependences into account we expect the pentaquark to be about 600–900 MeV heavier than the nucleon.

For the width calculations there are two principal differences to equation (19). First, the interaction Hamiltonian acquires an additional term

$$H_{\rm int}^{\rm SB} = \left(m_K^2 - m_\pi^2\right)d_{i\alpha\beta}D_{\gamma\alpha}D_{8\beta}\int d^3r z(r)\gamma(r)\widetilde{\xi}_\gamma(\vec{x}, t)\hat{x}_i. \tag{22}$$

The radial function $\gamma(r)$ is again given in terms of the chiral angle [4]. Second, X_Λ does not vanish as $N \to \infty$ and the R-matrix formalism is always two dimensional. Nevertheless, the large-N solution is always of the form (17) and the BSA phase shift is recovered. The width function turns to

$$\Gamma(\omega) = 2k\omega_0\left|\int_0^\infty r^2\, dr z(r)\left[2X_\Theta\lambda(r) + \frac{Y_\Theta}{\omega_0}\left(m_K^2 - m_\pi^2\right)\right]\overline{\eta}(r, \omega)\right|^2, \tag{23}$$

where X_Θ and $Y_\Theta = \sqrt{8N/3}\langle\Theta^+|d_{3\alpha\beta}D_{+\alpha}D_{8\beta}|n\rangle$ are to be computed in the RRA approach with full inclusion of flavor symmetry breaking effects.

This width function is shown (for $N = 3$) in figure 2 for Θ and its isovector partner Θ^*. The latter merely requires the appropriate modification of the matrix elements in equation (16). The k^3 behavior of the width function, as suggested by the model, equation (2), is reproduced only right above threshold, afterwards it levels off. Somewhat surprising, the width of the non-ground state pentaquark is smaller than that of the lowest lying pentaquark. Our particular model yields $\Gamma_\Theta \approx 40$ MeV and $\Gamma_{\Theta^*} \approx 20$ MeV. We note that there are certainly model ambiguities in these results.
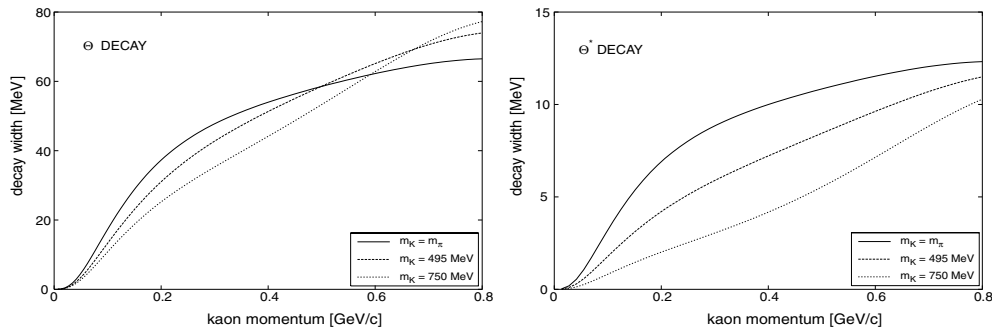
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Figure 2. Model prediction for the width, $\Gamma(\omega)$ of Θ^+ (left) and Θ^{*+} (right) for $N = 3$ for three values of the kaon mass. Note the unequal scales.

6. Conclusions

To exemplify the role that resonance exchanges play for the computation of scattering data in soliton models we have discussed KN scattering in the $S = +1$ channel which contains the potential [13] Θ^+ pentaquark, a state predicted as a flavor rotational excitation. Though the approach via small amplitude fluctuations suggests otherwise, the Θ emerges as a genuine resonance. A central result is the width function for $\Theta \to$ KN. In the flavor symmetric case it contains only a *single* collective coordinate operator and is thus very different from estimates that extract an effective Yukawa coupling from the axial current matrix element [3]. Since our approach matches the exact large-N result, we must conclude that those axial current scenarios are erroneous [14] and that soliton models unlikely predict very narrow pentaquarks.

Acknowledgments

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